

Overview of Particle Beam Optics Utilized in the Matrix, Envelope, and Tracking Codes: TRANSPORT, TRACE 3-D, and TURTLE

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Presentation Outline - Part I

Overview of Particle Beam Optics Utilized in the Matrix, Envelope, and Tracking Codes: TRANSPORT, TRACE 3-D, and TURTLE

1. **Basic Matrix Premise, Coordinates, Linear / Nonlinear Particle Optics, ...**
2. **Describing a Beam - Phase Space, Semi-Axes & Twiss Representations**
⇒ **Break**
3. **Equations of Motion: Drifts, Quads, Bends - Individual Particle Motion**
⇒ **Break**
4. **Introduction to the Beam Optics of TRANSPORT**
5. **Introduction to the Beam Optics of TRACE 3-D**
6. **Introduction to the Beam Optics of TURTLE**
7. **Summary**

Part II ⇒ **Will use the PBO Lab software in the class to illustrate concepts**

Presentation Outline - Part II

Overview of the PBO Lab Software with Examples and Sample Problems using TRANSPORT, TRACE 3-D, and TURTLE

8. **Overview of the Particle Beam Optics Laboratory (PBO Lab)**
Graphical Beamline Construction Kit
Interactive Tutorials
Built In First-Order Tools (Focusing & Bending)
9. **Application Modules: TRACE 3-D, TRANSPORT & TURTLE**
10. **Using TRACE 3-D, TRANSPORT & TURTLE to Solve Some Problems**

⇒ **You will use PBO Lab software in the class**

4. Introduction to TRANSPORT

TRANSPORT Overview

- **TRANSPORT**
 - **A First-, Second-, and (partial) Third-Order Code (FNAL, PBO-Lab)**
 - **Long History of Evolution - probably the oldest code still in use**
 - **Originally in "BALGOL" (~1960) - rewritten in FORTRAN (~1972)**
 - **Historically only included magnetic components (quads, bends, ...)**
 - **Numerous versions around in varied states of maintenance (errors)**
- **Applies R-, T-, and U-Matrices to Advance Beam Through Each Element**
 - **Beam Described by 1st & 2nd Moments of the Particle Distribution**
 - **Matrix Model of Optical Components**
 - **Include Models for Higher-Order Effects of Magnets**
 - **Vary Parameters (up to 20) with Many Options for Fit Constraints**
 - **Parameters Can Be Described / Defined by Formulas**
 - **Extensive Options for Output (text, plot files) Available**
- **Principal Use is for Transfer Line Design**
 - **Limited application to linacs, synchrotrons, storage rings, ...**
 - **No space charge modeling (except some early spin-off versions - 2D)**

4. Introduction to TRANSPORT (continued)

- **Several Magnetic Optical Elements Built into Standard Version**
 - **Drift Including All 2nd Order (but not 3rd order) Effects**
 - **Non-Bend Magnetic Elements (most through 3rd order):**
 - Quadrupole (3rd), Solenoid (2nd), Sextupole (3rd), Octupole (3rd)**
 - **Three Representations of Bending Magnet Elements (3rd order):**
 - Bend with Edges (pole face rotations, pole face curvature, ...)**
 - Sector Bend, or S-Bend, which has edge effects built-in**
 - Rectangular Bend, or R-Bend, which has edge effects built-in**
 - **Bending Magnets Can Include Multipoles (combined function bends)**
 - **Geometry Type Elements:**
 - Centroid Displacement / Reference Trajectory Shift**
 - Axial Rotation (Roll) about Beamline - PBO Lab Rotate Piece**
 - Increase in RMS Beam Properties**
- **Several Other Optical Elements (kicker, septum, plasma lens, ...)**
- **One Radiofrequency (RF) Built-in Optical Element in Standard Version**
 - **Traveling Wave Accelerator Model (1st order) for Electrons**
- **Can Use a Matrix (through 3rd order) Directly as an Element**

4. Introduction to TRANSPORT (continued)

- **Capable of Utilizing 2 Different Types of Input Files**
 - Old Style "Positional" Notation - input file is "entirely" numbers
 - MAD (Methodical Accelerator Description) with Keywords - PBO Lab
- **Can Accept Various Descriptions of Several Elements (MAD)**
 - PBO Lab supports this ("Green Dots")
- **PBO Lab TRANSPORT Has Additional Optical Elements Available**
 - RF Gap*, Thin Lens*, Quad Doublet*, QuadTriplet*
 - *TRACE 3-D Elements (see Section 5.)
 - Alias - Takes on the Identity of a Specified Element
- **PBO Lab TRANSPORT Will Automatically Display Requested Plots**
- **PBO Lab Can Create a Beam & Matrix Pieces using TRANSPORT Results**
 - Can Import TRACE 3-D Input Files from other TRACE 3-D versions*
 - Can Write TRACE 3-D Input Files for other TRACE 3-D versions*
 - *Assuming versions have some degree of compatibility!
- **Task 2 Utilized PBO Lab Version of TRANSPORT to:**
 - Add ElectroStatic Quad (through 3rd order)
 - Add Magnetic and ElectroStatic Quad 3rd order Fringe Field Integrals

4. Introduction to TRANSPORT (continued)

- **Input Beam Usually in Semi-Axes Representation (with Correlations)**
- **TRANSPORT Can Accept as Input a Twiss Representation of Beam**
 - Only Transverse (x-, y-) Phase Plane Twiss Parameters & Emittances
 - Longitudinal Beam Uses $\langle l^2 \rangle^{1/2}$ and $\langle \delta^2 \rangle^{1/2}$ (i.e. no correlation)
- **But TRANSPORT Will "Complain" If You Use the Twiss Parameters with certain elements that mix phase planes (solenoid, ...)**

To Complete This Introduction to TRANSPORT Let's Look At a Few of the Other Outputs that You Can Ask For

- **Any Matrix Can Be Printed at End (Exit) of Each Element**
 - Beam Matrix in the Reduced Form
 - R_{ij} Elements as a 6 x 6 Matrix, T_{ijk} as a Lower Half Matrix for Each i
 - U_{ijkl} as a Lower Half Matrix for Each i
- **Twiss Parameters (Beam in "Accelerator Notation") at Element Exits**
 - Includes Phase Advances & Emittances for Transverse Phase Planes
- **And Quite a Few Others \Rightarrow Data Can Also Be Output to a Plot File**
- **Essentially Anything that You Can Ask for as Output Can Also Be Used for Fitting Constraints \Rightarrow Extensive Fitting Capabilities**

4. Introduction to TRANSPORT (continued)

Some Useful R-Matrix Fitting Constraints

- **Trace of R-Matrix for stability in a periodic system:** $(1/2) |\text{Tr}[R]| \leq 1$
- **For point-to-point optics in the horizontal (x) direction:** $R_{12} = 0$
- **For parallel-to-parallel optics in the horizontal (x) direction:** $R_{21} = 0$
- **For parallel-to-point optics in the horizontal (x) direction:** $R_{11} = 0$
- **For point-to-parallel optics in the horizontal (x) direction:** $R_{22} = 0$
- **Similar conditions for the vertical (y) direction involving R_{yy} submatrix**
- **For achromatic optics in the horizontal (x) direction:** $R_{16} = R_{26} = 0$

Useful Beam (σ) Matrix Constraints

- **For a beam waist in the horizontal (x) direction:** $\alpha_x = 0$ or $r_{12} = 0$
 - **For beam size in the horizontal (x) direction:** $[\sigma_{11}]^{1/2} = X_{\text{size}}$
- ⇒ **TRANSPORT Documentation (pages 262-7) has more examples**

4. Introduction to TRANSPORT (continued)

- In 1st Order Calculation, TRANSPORT Advances $[q_i]_c$ and $[\sigma_{ij}]$ by

$$[q_i]_b = \mathbf{R}[q_i]_a \quad \text{and} \quad [\sigma_{ij}]_b = \mathbf{R}[\sigma_{ij}]_a \mathbf{R}^T$$

Note the "c"
for Centroid

- Computes $[q_i]_c$ and $[\sigma_{ij}]$ at the End (Exit) of Each Optical Element

⇒ To Get $[q_i]_c$ and $[\sigma_{ij}]$ Inside an Optical Element Must Split the Element

- In 2nd and 3rd Order TRANSPORT Also Advances $[q_i]_c$ and $[\sigma_{ij}]$, but the Formulas are More Complicated (involve T and U matrices)

⇒ Higher Order Effects on $[q_i]_c$ and $[\sigma_{ij}]$ are Essentially RMS Treatments

- Other "Ray Tracing" or "Tracking" Codes Advance each Particle's $[q_i]$

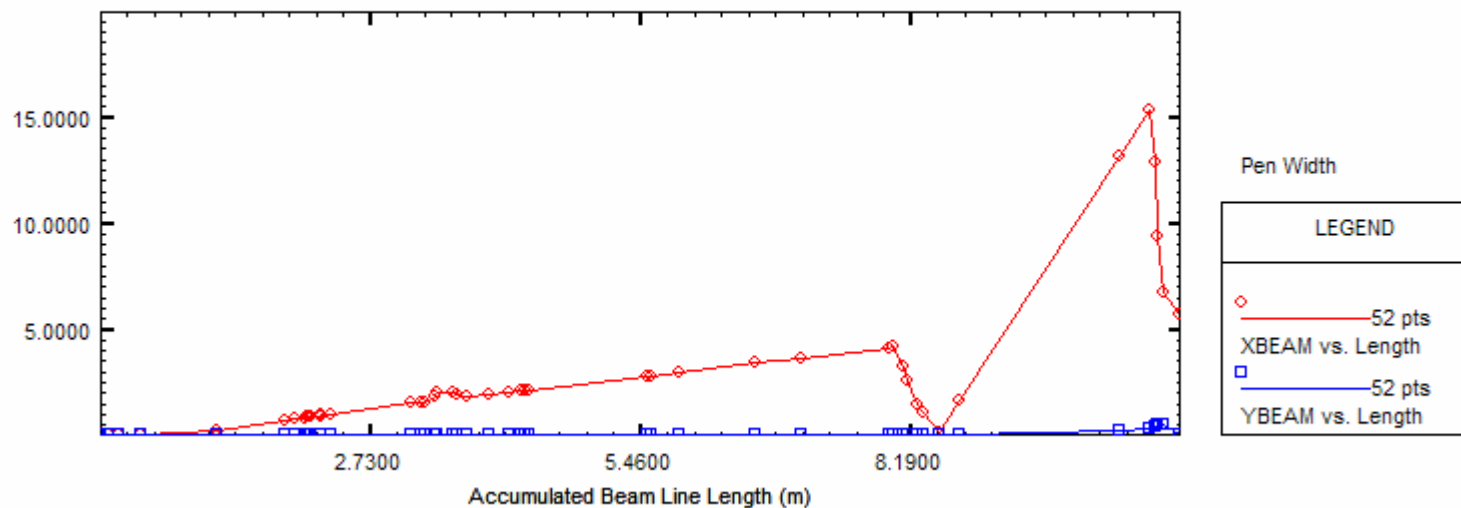
⇒ Ray Tracing Program TURTLE Tracks Individual Particle's $[q_i]$

- TRANSPORT is Useful for Eliminating/Reducing Higher Order Effects (e.g. higher-order fitting during the design process for a transfer line)

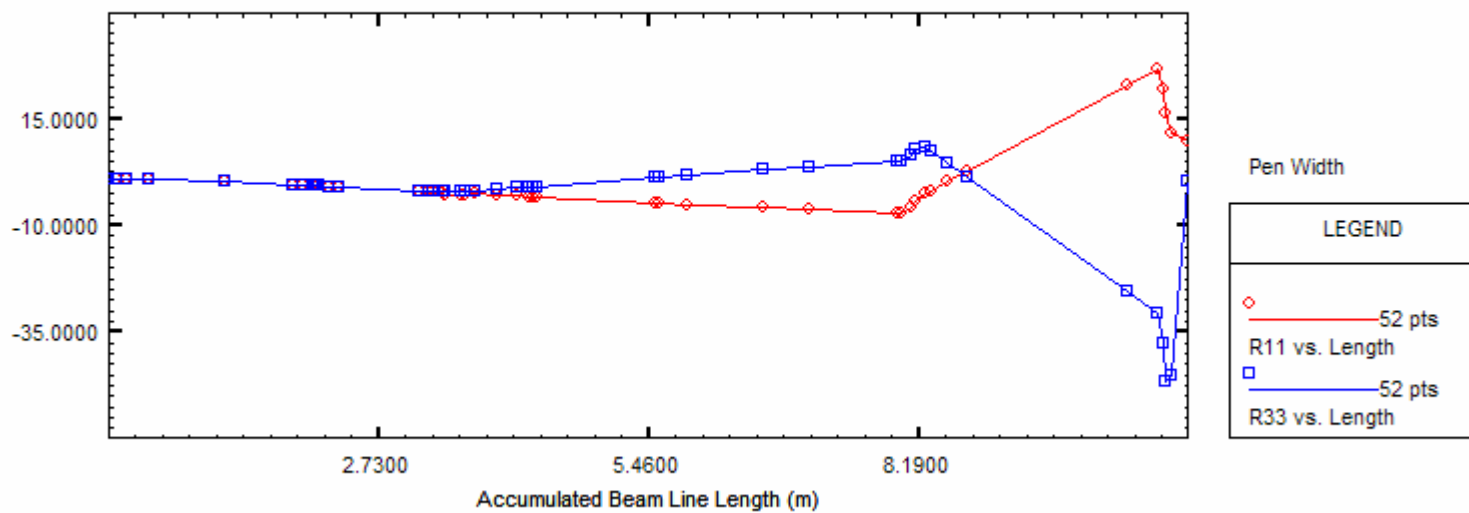
- Has Extensive Data Output for Plotting - PBO Lab provides the plots

4. Introduction to TRANSPORT (continued)

Beam Properties



Machine Properties



5. Introduction to TRACE 3-D

- **TRACE 3-D**
 - **Primarily a First-Order Code with a Space Charge Model**
 - **Evolved from an Earlier Two-Dimensional Code (TRACE)**
 - **Similar to an Early (LBNL) TRANSPORT Spin-Off**
 - **Includes several radiofrequency (RF) components**
- **Solves (Numerically “Integrates”) the Envelope Equations**
 - **Beam is an Ellipsoid in Three Dimensions - “Bunched”**
 - **Differential Matrix Model of Optical Components**
 - **Beam Envelopes Advanced in Steps, Using R-Matrices for Elements of Short Length, Δs**
 - **Space Charge Impulse Applied at Each Step**
 - **Can Include Models for Fringe Fields, Higher-Orders, Non-Linearities - But Only Computes Their Effect on the Second Moments of the Beam Distribution (σ Matrix)**
- **Principle Uses Are for Ion and (Low-Energy) Electron Beams**
 - **Especially for Radiofrequency Acceleration, Space Charge**
- **PBO-Lab Version Can Also Model ElectroStatic (ES) Elements**
 - **Einzel Lenses, ES Quadrupoles, ES Columns, ES Deflectors**

5. Introduction to TRACE 3-D (continued)

- **Initial Beam Usually Specified with 3-D Twiss (CS) Parameters**
 - **May Also Specify the Initial σ Matrix Directly**
- **6×6 σ Matrix Advanced, from Location j to $j+1$, through an Increment, $\Delta s = s_{j+1} - s_j$, Along the Central Trajectory:**

$$\sigma(j+1) = R(\Delta s) \sigma(j) R(\Delta s)^T$$

- **$R(\Delta s)$ is the First-Order Transfer Matrix for Optical Element of Length Δs**
- **At Each Increment, a Space Charge Impulse is Applied Using a Thin Lens R Matrix Based Upon 3-D Ellipsoid**
- **Since $R(\Delta s)$ is Computed At Each Increment j , Non-Linear & Non-Constant Fields Can be Modeled by Using $R(j, \Delta s)$**

5. Introduction to TRACE 3-D (continued)

- **Sixteen Built-in Optical Elements in Standard Version**
 - **Six are Same as TRANSPORT Elements:**
Drift, Quad, Solenoid, Bend, Edge, Rotate
 - **Three are “Compound” Magnet Elements:**
Symmetric Doublet*, Triplet*, PMQ with Fringe Fields
 - **Four are Radiofrequency Elements:**
RF Gap*, RFQ Cell, RF Cavity, Coupled Cavity Tank
 - **Thin Lens***
 - **Alias (Identical) - Takes on the Identity of a Specified Element**
 - **Special = Free Electron Laser (FEL) Wiggler**

- **PBO Lab TRACE 3-D Has Additional Optical Elements Available**
 - **2 Traveling Wave RF Accelerator Elements for Electron Linacs**
 - **Electrostatic (ES) Elements - These Are on Simulation Lab Computers**
3 Einzel Lenses, 3 Prisms (Deflectors), 2 DC Columns, 2 ES Quads
 - **TRANSPORT / MAD S-Bend and R-Bend Supported**

- ***PBO Lab TRANSPORT Supports Doublet, Triplet, RF Gap, Thin Lens**

5. Introduction to TRACE 3-D (continued)

Space Charge Model in TRACE 3-D

- **The Charge Density of a Uniformly Filled 3-D Ellipsoid is**

$$\rho(x,y,z) = \rho_o \Theta \left[1 - \left(\frac{x}{x_m} \right)^2 - \left(\frac{y}{y_m} \right)^2 - \left(\frac{z}{z_m} \right)^2 \right]$$

Where Θ is the Heaviside Step Function and

$$\rho_o = \frac{3Q}{4\pi x_m y_m z_m}$$

With Q Equal to the Total Charge in the Ellipsoid

- **The Three Semi-Axes of the Ellipsoid Are Computed from**

$$x_m = (\sigma_{11})^{1/2} \quad y_m = (\sigma_{33})^{1/2} \quad z_m = (\sigma_{55})^{1/2}$$

⇒ **Important to get σ_{55} correct, even for pure magnetic systems**

- **A Particle Will See an Electric Field Due to This Charge Density**
 - Inside the Ellipsoid, the Field is Linear in x, y, z
 - The Coefficients of the Linear Field Depend Upon x_m, y_m, z_m
 - TRACE 3-D Model Has No "Particles" Outside the Ellipsoid

5. Introduction to TRACE 3-D (continued)

Space Charge Model in TRACE 3-D (con't)

- **Particles Experience an Electric Field Due to $\rho(x,y,z)$**
Inside the Ellipsoid, this Field in the Beam Frame is Given by:

$$E_x = \frac{\rho_o}{\epsilon_o} \left[\frac{(y_m)}{(x_m+y_m)} \right] (1 - f) x$$

$$E_y = \frac{\rho_o}{\epsilon_o} \left[\frac{(x_m)}{(x_m+y_m)} \right] (1 - f) y$$

$$E_z = \frac{\rho_o}{\epsilon_o} f z$$

- **$f = f(p)$ is the Ellipsoidal *Form Factor* Which Depends Upon the Semi-Axes of the Ellipsoid (x_m, y_m, z_m) Through the Ratio p :**

$$p = \left[z_m / (x_m y_m)^{1/2} \right]$$

5. Introduction to TRACE 3-D (continued)

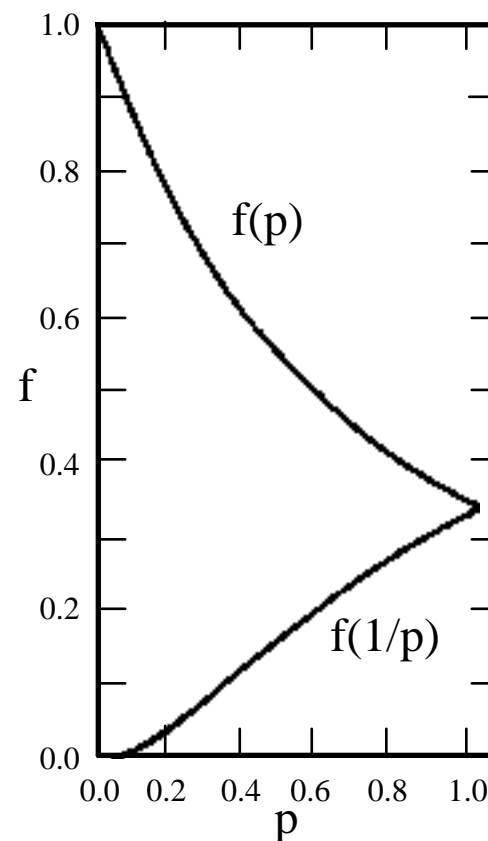
Space Charge Model in TRACE 3-D (con't)

Ellipsoidal Form Factor

- For $0 \leq p \leq \infty$, the Ellipsoidal Form Factor is $0 \leq f(p) \leq 1$
- When $p \cong 1$ (near spherical bunch) then $f(p) \cong 1/(3p)$

$$f(p) = \begin{cases} \frac{1}{1-p^2} - \frac{p}{(1-p^2)^{3/2}} \cos^{-1}(p) & , \text{ for } p < 1 ; \\ \frac{p \ln \cdot [p + \sqrt{p^2 - 1}]}{(p^2 - 1)^{3/2}} - \frac{1}{p^2 - 1} & , \text{ for } p > 1 . \end{cases}$$

$$f(1) = \frac{1}{3}$$



5. Introduction to TRACE 3-D (continued)

Space Charge Model in TRACE 3-D (con't)

- **For One Beam Bunch Passing a Point in the Beamline Every RF Cycle, the Total Charge is Related to the Beam Current I:**

$$Q = I/f = (\lambda/c)I$$

- **For Relativistic Beams with Kinetic Energy $W = (\gamma-1)mc^2$:**

$$(E_{x,y})_{\text{lab frame}} = (E_{x,y})_{\text{beam frame}} / \gamma$$

$$(Z_m)_{\text{lab frame}} = (Z_m)_{\text{beam frame}} / \gamma$$

- **Effective R Matrix is Equivalent to a 3-D Diverging Thin Lens**

$$R_{21} = -1/f_x = qe (\partial E_x / \partial x) \Delta s / (\gamma \beta^2 mc^2)$$

$$R_{43} = -1/f_y = qe (\partial E_y / \partial y) \Delta s / (\gamma \beta^2 mc^2)$$

$$R_{65} = -1/f_z = qe (\partial E_z / \partial z) \Delta s / (\gamma \beta^2 mc^2)$$

- **A Few Computational Details (Automated in TRACE 3-D)**
 - Ellipsoid May Be Tilted \Rightarrow Must Transform Coordinates
 - Calculation Accuracy \Rightarrow Elements at $\Delta s/2$, Some Adjust Δs

5. Introduction to TRACE 3-D (continued)

TRACE 3-D Matching Capabilities

- **“Matching” is TRACE 3-D Equivalent to TRANSPORT “Fitting”**
- **Fourteen (14) Matching Options in TRACE 3-D**
 - **Four (4) Find Twiss (C-S) Parameters for Matched Beams**
 - **One Varies Initial Beam Parameters to Produce Specified Twiss Parameters at the Output**
 - **Six (6) Vary (Match) Beamline Parameters to Produce Specified Twiss Parameters at the Output**
 - **Three (3) Vary Beamline Parameters to Produce Specified R Matrix Elements (for Overall Beamline)
Specified σ Matrix (Modified) Elements (at Output)
Specified Phase Advances μ_x, μ_y, μ_z (at Output)**
- **Number of Beamline Element Match (Vary) Parameters Limited to Six (6)**

5. Introduction to TRACE 3-D (continued)

TRACE 3-D Matching Capabilities (con't) Mismatch Factor

- **Useful to Have One Number (Figure of Merit) to Compare Two Ellipses**
- **One Measure of Comparison is the Mismatch Factor (MMF)**
 - **Two Ellipses (a and b) with Different Twiss Parameters in x Plane**
 - **Mismatch Factor Between Ellipses a and b Defined as**

$$\text{MMF}_x = \left[(1/2)(R_x + [(R_x^2 - 4)]^{1/2}) \right]^{1/2} - 1$$

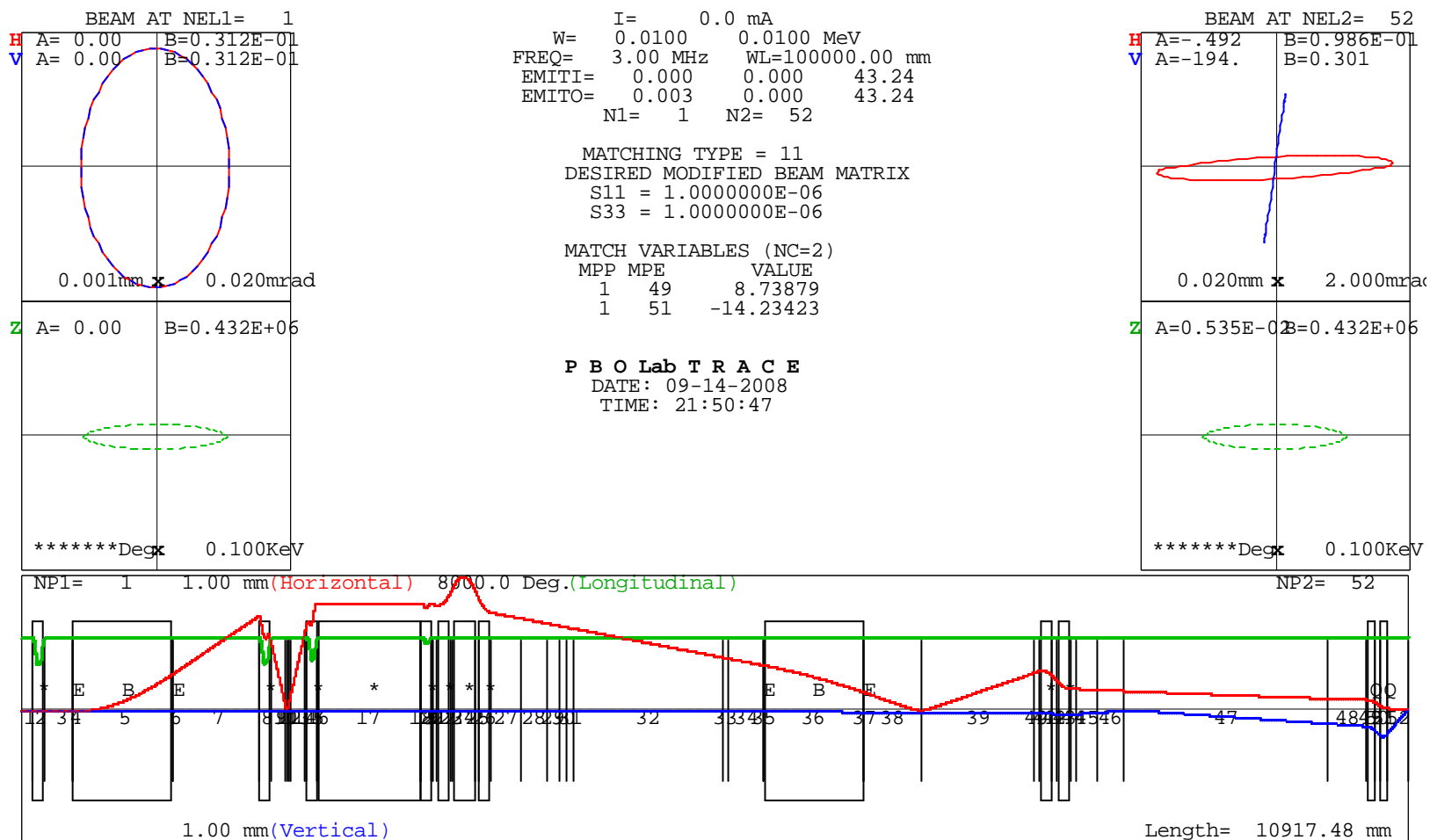
where $R_x = \beta_a \gamma_b + \gamma_a \beta_b - 2 \alpha_a \alpha_b$
 - **If Ellipses Are Identical (a=b):** $R_x = 2(\beta_a \gamma_a - \alpha_a^2) = 2$ & $\text{MMF}_x = 0$
 - **Different Ellipses** $\text{MMF}_x > 0$
- **Most TRACE 3-D Fitting Minimizes Mismatch Factors MMF_x , MMF_y , MMF_z**
- **Mismatch Factor defined by Twiss Parameters.**

5. Introduction to TRACE 3-D (continued)

Some Other TRACE 3-D Features

- **TRACE 3-D Can Run Beam in Reverse (Backward) Direction**
 - PBO-Lab Put “Initial” Beam at End of Beamline, “Final” Beam at Start
- **Supports Misalignment of Elements (computes beam centroid)**
- **Can Couple Elements Parameters to Match Parameters**
 - **k=+1 Coupling: Couple Parameter = Match Parameter**
 - **k=-1 Coupling: Couple Parameter = - Match Parameter, EXCEPT for Drift Lengths: Sum of 2 Drifts = Constant**
- **PBO Lab version of TRACE 3-D**
 - **Electrostatic (ES) Elements that can be used by TRACE 3-D**
 - **Can Import TRACE 3-D Input Files from other TRACE 3-D versions***
 - **Can Write TRACE 3-D Input Files for other TRACE 3-D versions***
 - *Assuming versions have some degree of compatibility!
- **Display Options Limited: Profiles and Phase Space Plots**
 - **Can Overlay ("Trace on Background") Profiles for Comparison**

5. Introduction to TRACE 3-D (continued)



5. Introduction to TRACE 3-D (continued)

Quick Comparison of Main Features

TRANSPORT

1st, 2nd, 3rd Order Optics
No Space Charge (low current)
Outputs Data Files for Post Plotting
Up to 20 Vary Parameters
Formula Coupling
Numerous Fitting Conditions
Magnetic Elements, 1 RF
RMS Beam Properties
Transfer Lines a Main Application
40+ Years of Usage
Source Available from FNAL
(other versions exist)

TRACE 3-D

1st Order Optics +
Linear Space Charge (high current)
Graphic Display of Beamline
Up to 6 Match Parameters
k = ± 1 Coupling
14 Matching (Fitting) Conditions
Magnetic & RF Elements
(5)^{1/2} RMS Beam Properties
RF Linacs a Main Application
30+ Years of Usage
Executable Available from LANL
(other versions exist)

PBO Lab Versions Have a Number of Additional Capabilities

6. Introduction to TURTLE

- **Acronym: "Trace Unlimited Rays Through Lumped Elements"**
- **Companion Code to TRANSPORT**
- **"Performance Code" Rather Than a "Design Code"**
 - **Calculates the 6-D coordinates of macroparticles through beamline**
 - **Same 6-D coordinates as TRANSPORT: $(q_i)=(x,x',y,y',l,\delta)$**
 - **Can routinely run 1,000 - 10,000+ particles on a PC**
 - **No parameter variation, fitting, matching, ...**
- **Can examine effects not modeled in TRANSPORT**
 - **Three distributions in standard TURTLE:
Gaussian, Rectangular, Uniform**
 - **Beam loss on apertures and slits**
 - **Non elliptical beams - either as input, or created in a beamline**
 - **Can add phase space "cuts" to distributions along beamline**
- **Output Used by Some Radiation Transport Codes - Shielding**

6. Introduction to TURTLE (continued)

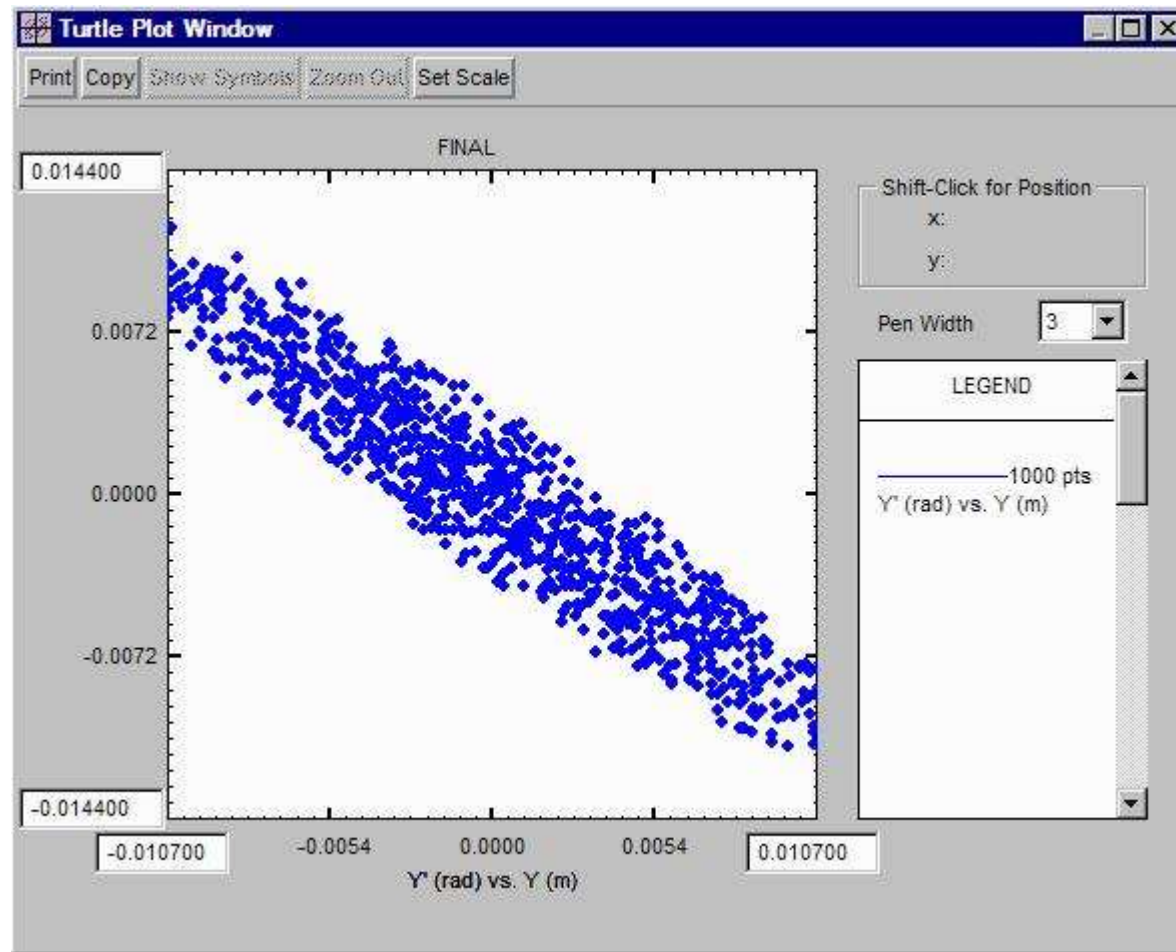
- **TURTLE is complimentary program to TRANSPORT**
 - Both use same description of beamline and elements
 - 1st, 2nd, 3rd order calculated in both
 - Use the same 6-D phase space coordinates
 - ⇒ **Use TRANSPORT to get a design; use TURTLE to evaluate it**
- **A special version, called DECAY-TURTLE, for radioactive beams**
 - Tracks particles that have a finite lifetime
 - Primary particles (parents) decay randomly during beamline passage according to their lifetime
 - Tracks the decay product particles (daughters) thru beamline
 - User specifies parent lifetime, masses and charges of daughters
 - Transparency of apertures and slits can be particle dependent
- **PBO-Lab provides TURTLE support for the same "non-TRANSPORT" optics elements that PBO-Lab makes available for TRANSPORT**
 - TRACE 3-D Elements: Doublet, Triplet, RF Gap, Thin Lens
- **Task 2 Utilized PBO Lab Version of TRANSPORT to:**
 - Add ElectroStatic Quad (through 3rd order)
 - Add Magnetic and ElectroStatic Quad 3rd order Fringe Field Integrals

6. Introduction to TURTLE (continued)

- **Generates Data for 2 Types of Plots**
 - **Scatter Plots of Phase Space Data (2-D Plot) e.g. x' versus x**
 - **Histograms of Data (1-D Plot) e.g. # particle is at bin location x**
- **Any of the 6-D Phase Space Variables Can be Used x, x', y, y', l, δ**
 - **36 Possible Scatter Plots**
 - **6 Possible Histograms**
- **Plots are Text Based - "Line Printer" (without a post processor)**
- **Multiple Locations in Beam Line - User Specified Plots at Each**
- **Apertures (on magnets, etc,) Can Be Ignored or Included (Particle Loss)**
- **DECAY-TURTLE Version Has Above Capabilities for Decay Particles**
 - **But DECAY-TURTLE **does not fully support 3rd order****

6. Introduction to TURTLE (continued)

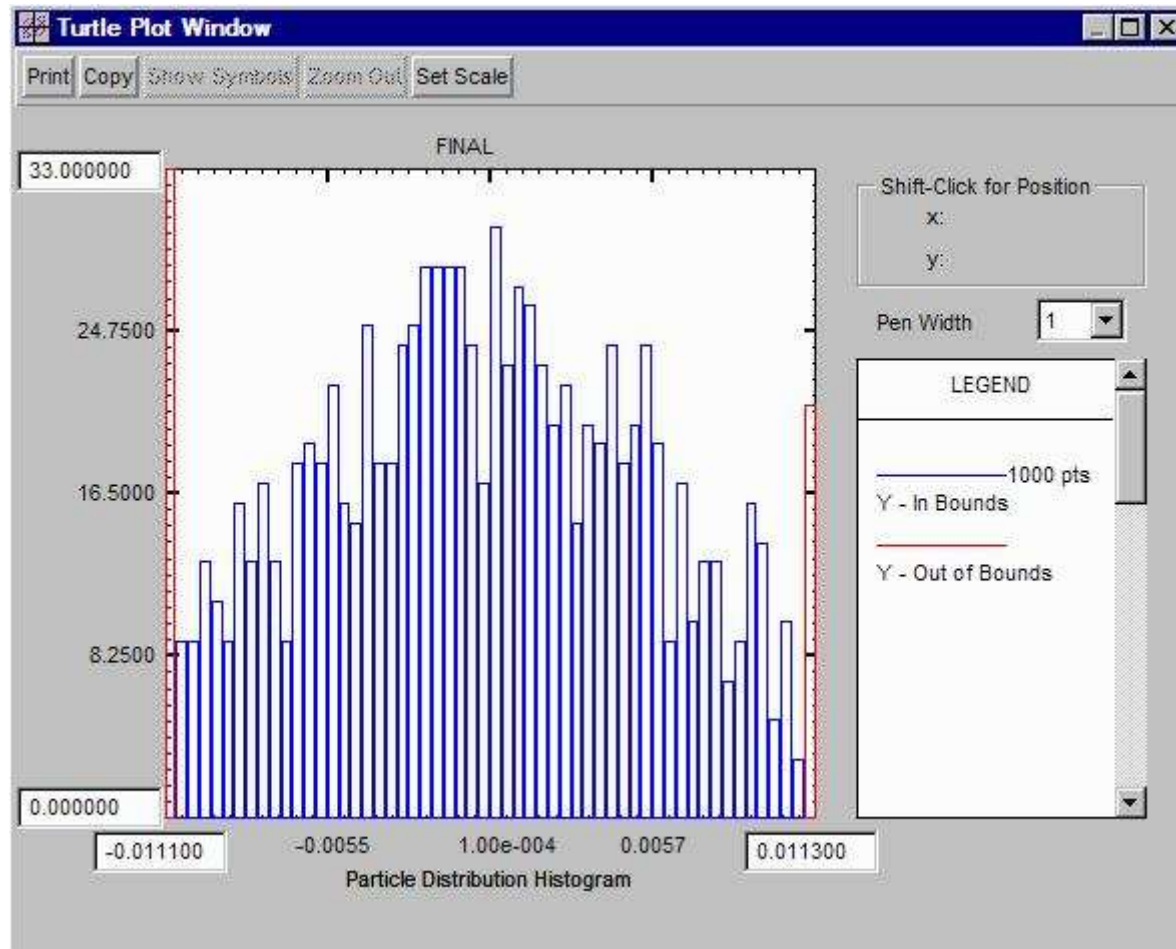
Typical Scatter Plot (PBO Lab postprocessor plot)



- **PBO-Lab uses same (default) color assignments for all codes**
- **Provide for "Auto-Scaling" of Plots - Useful for Initial Setup**
- **Can Adjust Scales After a Run- Unlimited "Zoom" Capability**

6. Introduction to TURTLE (continued)

Typical Histogram (PBO Lab postprocessor plot)



- PBO-Lab plots provide data visualization & document preparation tools
- Print, Copy, scale adjustment, ...
- Need to look at TURTLE (text) output for selected quantitative data

7. Summary of Part I

- **Overview of Coordinate Systems and Basic Matrix Descriptions**
- **Relationship Between Semi-Axes and Twiss Beam Description**
- **Overview of Drift, Quad, and Bend Equations of Motion & Matrix Solutions**
- **Guide to Fitting Constraints (Point-to-Point, etc.)**
- **Summary of Primary TRANSPORT, TRACE 3-D & TURTLE Capabilities**

Part II ⇒ **Will use the PBO Lab software in the class to illustrate concepts**
Complete some typical computations with the codes in the class